Survey of the Research on Intuitionistic Fuzzy Sets

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Abstract. This paper presents the development of the theory of Intuitionistic Fuzzy Sets (IFSs).

Introduction

In 1983, K. Atanassov proposed a generalisation of the notion of fuzzy set, known as Intuitionistic Fuzzy Set (IFS). He introduced a new component, degree of nonmembership, in the definition of these sets and studied the properties of the new object so defined. Since then, a great number of theoretical and practical results appeared in the area of IFSs. There are numerical applications of IFSs in various areas of computer science, for example, in artificial intelligence, as well as in medicine, chemistry, economics, astronomy, etc. Researchers in Australia, Belgium, Bulgaria, China, India, Poland, Romania, South Korea, Spain, Turkey, USA and many other countries have advanced and promoted the theory of IFSs. The international journal "Notes on Intuitionistic Fuzzy Sets" is regularly issued since 1995. The annual International conferences on IFS have been held for the last five years.

Below we present theoretical and practical results on IFSs, grouped in several main parts: foundations of IFSs; IF geometry and analysis; IF logics; IF approach to artificial intelligence; IF generalized nets; applications of IFSs.

The complete bibliography on IFS now contains more than 500 items. Due to lack of space, they are not included in this paper, but can be found at the Internet address http://clbme.bol.bg/ifs.

Foundations of IFSs

Definition of the concept of Intuitionistic Fuzzy Set (IFS) and its modifications and properties

Fuzzy sets were introduced by L. A. Zadeh in 1965. One generalisation of the notion of fuzzy sets was proposed by K. Atanassov in the beginning of 1983 and presented before the Seventh Scientific Session of ITKR, Sofia, June 1983. [1] In addition to the degree of membership known from fuzzy sets, here a new degree is introduced, called degree of non–membership, with the requirement that their sum be less than or equal to 1. The complement of the two degrees to 1 is regarded as a degree of uncertainty. This new extension of the concept of fuzzy sets was named intuitionistic fuzzy set (IFS). More details can be found in [2, 3].

The name of intuitionistic fuzzy sets is due to George Gargov, with the motivation that their fuzzification denies the law of the excluded middle—one of the main ideas of intuitionism.

Later in 1983 it turned out that the new sets allow for the definition of operators which are generalizations of the modal operators of necessity and possibility, which was the first serious result connecting IFS with classical logic and set theory.

Later, K. Atanassov and S. Stoeva further generalised IFSs to define intuitionistic L–fuzzy sets, where

L stands for some lattice coupled with an order–reversing operation.

Together with L. Atanassova they gave an example of a genuine IFS (an IFS which is not a fuzzy set). There is an analogous example for intuitionistic L–fuzzy sets.

A further generalisation of IFSs is the concept of interval-valued IFSs. G. Gargov and K. Atanassov showed that the latter are equivalent to IFSs.

G. Deschrijver and E. Kerre also showed that IFSs are L-fuzzy sets w.r.t. the lattice $(L^*, \leq_{L^*})$ defined as $L^* = \{(x_1, x_2) \in [0,1]^2 \mid x_1 + x_2 \leq 1\}$, $(x_1, x_2) \leq_{L^*} (y_1, y_2) \Leftrightarrow x_1 \leq y_1 \wedge x_2 \geq y_2$. Hence an IFS on X can be seen as an $X - L^*$ mapping. This approach offers greater flexibility in calculating with membership and non-membership degrees, since the pair formed by them is an element of L^*. Also, in some cases, more compact formulas are obtained.

Operations and relations over IFS. Algebraic research in the frameworks of the IFS theory

The first operations and relations defined over IFSs took as a point of departure respective definitions of relations and operations over fuzzy sets and extending them. Conversely, relations and operations over fuzzy sets turn out to be particular cases of the new definitions.

Later, new operations were proposed, some of which cannot be transferred to the case of fuzzy sets.

General versions of operations are defined, i.e. operations generalising more than one corresponding operation. Their basic properties are studied.

Algebraic research within IFS theory is aimed at defining IF subgroups, constructing the category IFuz of IFS and other related categories. IF filters and ideals of lattices were also introduced.

Operators over IFS and their properties

Various operators are defined on IFSs. Two of them are analogous to the modal logic operators of necessity and possibility and have no counterparts in the ordinary fuzzy set theory. These operators are subsumed in several new operators which in turn have no analogs in modal logic as well.

It was shown that IF necessity and possibility operators transform an IFS to a fuzzy set; they are essentially non–trivial in an IFS setting, while on fuzzy sets their result is trivial.

Another type of operators defined on IFSs are analogs to the topological operators of closure and interior.

Similarly to fuzzy set theory, level operators on IFS have also been proposed.

The relationships between the operators, as well as between operators and operations are investigated.

Geometrical interpretations of IFSs

There are geometrical interpretation of the elements of IFSs, IF objects, IF logical objects, interval valued IFSs and operations over IFSs.

Two types of interpretations exist so far: showing functional dependency (as in the case of fuzzy sets), now with both degrees—of membership and of non–membership; and properly geometrical interpretations: representability of IF degrees either as coordinates of points in a triangle, or as triangle angles or sides. As a whole, geometrical interpretations of IFSs are planar, but an interpretation on a sphere has also been proposed.

IF geometry, IF Analysis and IF Topology

IF numbers

There are different aproaches to define IF numbers. T. Buhaescu, P. Burillo, H. Bustince, V. Mohedano and M. Nikolova have worked in this area. In the definition of an IF number, we can drop the requirement that the degree of membership reaches one. Instead, it can be replaced by the weaker condition that the maximum of the degree of membership be greater than the corresponding degree of non-membership. This is related to IF tautology notion.

IF analysis and topology

Concepts of convexity and concavoconvexity for IFSs and temporal IFSs are introduced. The concept of intuitionistic (fuzzy) measure is defined. To this end, A. I. Ban introduces the limit of a sequence of IFSs. With the help of an abstract integral he gives a family of intuitionistic entropies introduced by Burillo and Bustince. It is proven that certain intuitionistic entropies are IF measures.

Active research on IF topology is carried out by the group headed by D. Coker, S. Samanta, R. Biswas; among their results is the notion of IF metric space and the topology of interval-valued IFSs. The solution concept for a semi–linear equation with IF parameters is also studied. L. Huawen and S. Wenqing have proposed another definition of IF topology.

IF Logics

The definitions in IF logic generally follow those of IFS. Operations, quantifiers, and modal operators are defined. Relations between the quantifiers and the modal type of operators are studied. The notion of tautology is generalised to IF tautology—a formula whose degree of truth is greater than or equal to its degree of falsity under every interpretation. It is checked that the axioms of various axiomatic systems can be proven as IF tautologies within IF setting. First steps are made towards development of IF temporal logic. Norms and metrics over IF logics are defined. Rules of inference are also studied.

IF propositional and predicate calculus

Two variants of IF propositional and predicate calculus have been proposed, and a completeness theorem is proved. Research has been done on conjunctions, disjunctions and implications of the IF logic, IF Modus Ponens, and IF deductive closure of a set of formulas. Implication can be defined in different ways in an IF setting, which affect the notion of IF tautology and Modus Ponens.

IF modal and temporal logics

Two variants of IF modal logic are proposed. Operators of modal type in IF modal logic are defined.

Temporal operators have been defined and their properties are studied.

Connections between IF logics and other logical systems

An IF model of the axioms of the paraconsistent set theory NF_1 is developed. IF interpretations are given of the conditional logic VW, Kolmogorov's axioms, Lukasiewicz's axioms, Kun's axiom, Meredith's axiom and others. Hauber's law is proven to be an IF tautology. Other problems are also considered.

IF Approach to Artificial Intelligence

Decision making and machine learning

E. Szmidt and J. Kacprzyk extend the classical Bellman and Zadeh's general approach to decision making under fuzziness, originally termed decision making in a fuzzy environment, to the case of IFS. Fuzzy routing models in terms of IFSs are considered.

Neural networks and pattern recognition

IF versions are proposed for one of the basic statistical nonparametrical methods, the k-nearest neighbour method. Classification problems in AI are discussed. Coupling IFSs with training Neural Nets is also studied.

Expert systems, machine reasoning, logic programming and IF Prolog

A notion of IF database is introduced. Using IF logic, a construction of Investment Expert System has been studied. IF prolog, IF constraint logic programming, IF expert systems and other topics are proposed and investigated.

IF relational calculus

In IF relational calculus, IF relations describe incomplete and/or uncertain information on the values a variable may assume. The added value of IF relations lies in their ability to capture varying degrees of reliability of information. Knowledge manipulation is done by the IF extension of the Combination–Projection Principle, a special instance of which, the IF Compositional Rule of Inference, applies to relational knowledge expressed as if–then rules.

Knowledge representation

Knowledge representation by IF frames has been proposed. Basic definitions and properties of formal concept analysis have been studied in the IF setting.

IFSs and interval data analysis

It was shown that any collection of numerical interval data can be interpreted as points in the IFS triangle. Therefore, one can work with these points, which is significantly simpler than working directly with interval data.

IF Generalized Nets

Generalized Nets (GN) are an extension of Petri nets, including as particular cases all other Petri net modifications. They contain tokens which enter with initial characteristics, and move throughout the net retaining their previous characteristics. Every GN transition is assigned a predicate matrix determining which tokens can pass from an input to an output place of the transition. Various fuzzifications of GNs are described in [2]; this is also the first publication to relate the concept of Petri nets with fuzziness.

K. Atanassov has defined four types of IF GNs as well as algorithms of their functioning. In some of them, transition condition predicates are evaluated by IF values; in others, in addition to that, tokens are replaced by quantities flowing within the net.

IF GNs were applied to describe functioning and results of the work of expert systems, machine learning, image and pattern recognition and generation, flexible manufacturing systems, robotics etc.

Applications of IFSs

Applications in medicine

There are applications of IFS in medical diagnosis, of IF logic in decision making in medicine, and also IF generalized net model of health unit activities, on the one hand, and human body activities, on the other.

Applications in optimization

P. Angelov has solved some optimization problems by means of IFSs and also has worked on optimization in an IF environment.

Research has been carried out on IF equations, inequalities, optimization and other problems.

Applications in chemical engineering and metallurgy

There are up to now a number of applications of IFSs in this area, for example: a method for simulation of complex technological system by use of IF generalized nets, an IF generalized net approach for optimal scheduling of iron ore delivering and discharge and blending yards creation.

Applications in economics

IFS approach in credit risk assessment has been proposed. D. Dimitrov has developed IF models of oligopolistic/monopolistic market as well as IF-aided decision making in economics.

Other applications

There are also IF generalized nets models of gravity field, in astronomy, sociology, biology, bioprocess engineering, musicology, computer hardware.

IF (Abstract) Systems

IF abstract systems and IF abstract systems with properties are defined and a lot of their properties are studied. Some applications in AI are discussed.

IF Graphs

A first step to describe a theory of the IF graphs and temporal IF graphs is made. Application of IF graphs and IF relation methods are also developed. The index matrix representation of IF graphs is described.

Bibliographical Remarks

Several reviews of the research on IFSs have been published so far. Here they are essentially extended and updated. The number of publications in the area has increased significantly in the recent years. A version of this paper containing the complete bibliography on IFSs can be found at the Internet address: *http://clbme.bol.bg/ifs*

References

[1] Atanassov, K. Intuitionistic fuzzy sets. Seventh Scientific Session of ITKR, Sofia, June 1983; Deposed in CINTI, 1983.

[2] Atanassov, K. Intuitionistic Fuzzy Sets. Springer-Verlag, Heidelberg, 1999.

[3] Atanassov K., Intuitionistic fuzzy sets, Fuzzy sets and Systems Vol. 20 (1986), No. 1, 87-96.