Linguistic Hedges: a Quantifier Based Approach

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Abstract. We present an entirely new approach for the representation of intensifying and weakening linguistic hedges in fuzzy set theory, which is primarily based on a crisp ordering relation associated with the term that is modified, as well as on a fuzzy quantifier. With this technique we can generate membership functions for both atomic and modified linguistic terms. We prove that our model respects semantic entailment and we show that it surpasses traditional approaches, such as powering and shifting modifiers, on the intuitive level and on the level of applicability.

1 Introduction

The success of fuzzy expert systems is greatly due to their ability to represent and handle vague information expressed by means of linguistic terms in facts and (IF-THEN) rules. Indeed, this tends to make such systems very compact and tolerant for the imprecision and incompleteness that often afflicts our knowledge of the real world. Furthermore, their input, output and inference mechanism can be more easily understood by humans, who's most popular daily means of communication and reasoning is, after all, natural language. In fuzzy systems, each linguistic term is represented by a fuzzy set on a suitable universe X [14]. A fuzzy set A on X is a $X - [0,1]$ mapping, also called the membership function of A, such that for all x in X, $A(x)$ is the membership degree of x in A. In this paper we will use the same notation to denote a fuzzy set A and the term A represented by it.

One of the most crucial and often difficult tasks in developing a fuzzy expert system is the construction of the membership functions for the linguistic terms involved. Fortunately these terms have a specific structure [15] which allows to partially automate this task by computing the membership functions of composed linguistic terms from those of atomic ones. In this context, an atomic linguistic term is an adjective (e.g. expensive). Composed terms are generated by applying either a linguistic modifier to a term (e.g. very expensive), negating a term (e.g. not very expensive) or by combining terms by means of a connective (e.g. not very expensive or sophisticated, nice and easy). If A and B are fuzzy sets on X, the terms A and B, A or B and not A can respectively be modelled by the Zadeh-intersection, the Zadeh-union and the complement, defined as follows

$$\begin{aligned} A \cap B(x) &= \min(A(x), B(x)) \\ A \cup B(x) &= \max(A(x), B(x)) \\ co\ A(x) &= 1 - A(x) \end{aligned}$$

for all x in X. It is even more common to model them by $\mathcal{T}$-intersection, $\mathcal{S}$-union and $\mathcal{N}$-complement based on a triangular norm $\mathcal{T}$, a triangular conorm $\mathcal{S}$ and a negator $\mathcal{N}$; the above mentioned operations are just special cases of this more general approach. In this paper however we will focus on linguistic modifiers (also called linguistic hedges). Since the origin of fuzzy set theory, many researchers have paid attention to the representation of these adverbs, probably because they allow for the generation of many modified terms from existing ones. The first serious attempt was sketched by Zadeh, as early as 1972 [15]. For the representation of very A and more or less A, he defined the concentration and dilation operator (also called powering modifiers) based on a simple involution, i.e. very $A(x) = A(x)^2$ and more or less $A(x) = A(x)^{0.5}$, for all x in X. One can easily verify that in this representation

$$\text{very } A(x) \leq A(x) \leq \text{more or less } A(x)$$

for all x in X. The satisfaction of this so-called semantical entailment [11] (very A is a subset of A, which in turn is a subset of more or less A) can be considered as a prerequisite for any representation, since very is an intensifying modifier, while more or less has a weakening effect. The best known shortcomings of Zadeh's approach, pointed out in e.g. [8, 9, 11], are that they keep the kernel and the support, which are defined as

$$\begin{aligned} \ker A &= \{x | x \in X \wedge A(x) = 1\} \\ \text{supp } A &= \{x | x \in X \wedge A(x) > 0\} \end{aligned}$$

As a consequence, this representation cannot distinguish between being A to degree 1 and being very A to degree 1. One might feel however that a person of 80 years is old to degree 1 but very old only to a lower degree (e.g. 0.7), but this cannot be modelled by means of powering modifiers. The shifting modifiers, informally suggested by Lakoff [11] and more formally developed later on [3, 8, 9], do not have this shortcoming, but it is a serious drawback that they cannot be applied straightforwardly to all kinds of membership functions in the same way, and hence sometimes require artificial tricks. Many representations developed in the same period are afflicted with very similar disadvantages as the powering and shifting modifiers (we refer to [10] for an overview). We believe these kinds of shortcomings on the level of intuition and the level of applicability are due to the fact that these modifiers are only technical tools, lacking inherent meaning.

In fact it was not until the second half of the 1990's that new models with a clear semantics started to surface. In the horizon approach [12] the semantics is derived from the concepts of horizon and visibility [5]. It is one of the few techniques in which the representation for both atomic and modified terms is generated from within the model (which can be a constraint if one prefers to modify membership functions for atomic terms obtained elsewhere). In the fuzzy relational based model [6], the semantics is retrieved from the context. A characteristic of the traditional approaches is that they do not really look at the context: to determine the degree to which x is very A, the concentration operator for instance only looks at the degree to which x is A and ignores the objects in the context of x. By bringing a fuzzy relation into action however, the membership degrees of the elements related to x can also be taken into account to some extent. Depending on the nature of the fuzzy relation, different kind of linguistic modifiers can be modelled within the same framework: an ordering relation gives rise to ordering-based modifiers such as at most and at least [2], while a resemblance relation can be used to represent both weakening (such as roughly and more or less) and intensifying hedges (such as very and extremely) [4]. Informal starting points for the latter two are observations such as "a person is more or less old if he resembles an old person" and "a person is very old if everybody he resembles is old". Translating all components of these observations into their fuzzy set theoretical counterparts, results on the formal level in the use of the direct image (related to the compositional rule of inference) and superdirect image of the fuzzy sets being modified under the fuzzy relations that describe the context.

In this paper we will once again attempt the technique of transforming natural language statements, describing the meaning of (modified) terms, into their mathematical counterparts. This time however the starting point will not be our intuition, but an analysis made by Wheeler in 1972 [13]. Putting all pieces of the puzzle together in the right way gives rise to a stunningly elegant, computationally efficient and semantically very comprehensable representation of both (!) atomic terms and modified terms. The key notion is that of a quantifier, which is not surprising since Wheeler's goal was to reveal that English is a first-order language at some level of analysis. Besides this, relations are once again brought into action to model the context. Most remarkable (compared to the fuzzy relational based approach) is however that crisp ordering relations prove to be very useful to model intensifying and weakening hedges. Since Wheeler describes the meaning of rather and very, we will study the representation of precisely these two hedges. However before presenting the model for modified terms (Section 4), we go into the representation of atomic terms (Section 3) after the necessary preliminaries (Section 2).

2 Preliminaries

Throughout this paper, let X denote a finite universe of discourse (i.e. a non-empty set containing a finite number of objects we want to say something about). In the following, the class of all fuzzy sets on X will be denoted by $\mathcal{F}(X)$. A fuzzy set A takes membership values in the real unit interval $[0,1]$. If all membership values of A belong to $\{0,1\}$, A is called a crisp set. In this case, the notation $A(x) = 1$ corresponds to $x \in A$, while $A(x) = 0$ is the same as $x \notin A$. A fuzzy relation R on X is a fuzzy set on $X \times X$. If the relation R is crisp, we denote $(x, y) \in R$ also by xRy. A useful concept concerning fuzzy relations is that of foreset.

Definition 1 (Foreset). [1] Let R be a fuzzy relation on X and y in X. The R-foreset of y is the fuzzy set Ry defined by $Ry(x) = R(x, y)$, for all x in X.

In other words the R-foreset of y is the fuzzy set of objects related to y. Furthermore we need the concept of inclusion of fuzzy set, as a means to express the mathematical couterpart of semantic entailment.