On the Meaning of Noise Annoyance Modifiers: A Fuzzy Set Theoretical Approach

D. Botteldooren, A. Verkeyn

Acoustics group, Department of Information Technology, Ghent University St. Pietersnieuwstraat 41, 9000 Gent, Belgium

C. Cornelis, M. De Cock

Fuzziness and Uncertainty Modeling group, Department of Applied Mathematics and Computer Science, Ghent University, Krijgslaan 281 (S9), 9000 Gent, Belgium

Summary

This paper reports on a fuzzy analysis of information gathered by many colleagues on the precise meaning of noise annoyance modifiers in 9 different languages. It is shown how fuzzy set theory can help us to construct a mathematical background for translating these modifiers between the languages concerned. A second goal of annoyance modifier research is to define labels to be used in noise annoyance surveys in order to obtain accurate and comparable results. Similarity measures used to compare fuzzy sets associated with verbal descriptors of annoyance levels indicate to what extent previously proposed labels [1] match between the languages considered. An ideal language from the fuzzy point of view where a continuous annoyance scale is exactly divided into n equal parts is translated to these natural languages and results in an alternative selection of labels that are better suited for fuzzy calculus. In general this selection of labels corresponds quite well with the set proposed in [1] which is rather surprising since the fuzzy set approach lacks most of the human input used in the ICBEN selection procedure.

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1. Introduction

Noise annoyance, although vague in concept, has been used as an indicator of the adverse effect of noise on man. A vast amount of knowledge has been gathered in social surveys using various types of questionnaires. Meta-analyses that were proposed to extract more general dosage response relations [2, 3] have been confronted with different annoyance scales both verbal and non-verbal. Verbal scales introduce the additional complication of language. Words used in surveys in different language regions do not necessarily match exactly to words in another language so no "exact" translation is possible.

In 1993 the Community Response to Noise Team (Team 6) of the International Commission on the Biological Effects of Noise (ICBEN) therefore developed a program to facilitate comparisons between socio-acoustic surveys. Their work included a standardized research project that chose the labels for the answers to a 5-point verbal scale. The procedure started with selecting a pool of 21 modifiers (adverbs) of annoyance. These terms were then presented to a mixture of university students and employees of technical firms. The average age was about 35 years, but

varied from 19 to 44 for different study sites. After providing some background information the subjects completed the questionnaire by performing the following four tasks to evaluate the 21 words:

Task 1: Subjects placed each word in one of nine groups ranked from "no annoyance" to "the most annoyance you can imagine."

Task 2: Subjects indicated the intensity associated with each word by placing the word on its own 10-cm line that extended from "No/lowest degree of annoyance" to "Highest degree of annoyance."

Task 3: Subjects selected one preferred word for each of the scale points by first choosing a word "that you would be most likely to use" for the "greatest amount of bother or annoyance you might feel" and then expressing a preference for the three words that should complete the remaining three points on a 5- point scale. (The lowest point was predetermined.)

Task 4: Same as 3 but for a 4-point preference question. For both the 4- and 5-point preference questions subjects were instructed to choose words that "people would normally use when talking". Subjects were instructed to select words that were "equally spaced" between "not at all annoyed" and the previously chosen high annoyance word. The questionnaires were completed by 1 754 subjects at over 25 sites in 12 countries in nine languages

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(Dutch/Flemish = 93, English = 231, French = 45, German = 61, Hungarian = 47, Japanese = 1102, Norwegian = 56, Spanish = 59, Turkish = 60). Conclusions on preferred annoyance scale labels were finally drawn on the basis of the results of tasks 2 to 4. An extensive report on this research can be found in [1, 4].

In fuzzy rule based systems [5] it is common practice to mathematically represent a linguistic term by a fuzzy set on a predefined universe U, which is a mapping $U \rightarrow [0, 1]$ called the membership function. A whole suite of operators (mainly generalizations of operators known from classical logic) is then available to perform operations on these membership functions (see Appendix A1). Since the membership function can be interpreted as representing the concept underlying the linguistic term used to describe it [6], the similarity between membership functions of linguistic terms in different languages can be used for translation provided a suitable universe U can be found [7]. In the case of noise annoyance modifiers such a universe is available as the continuous annoyance level line used in the experimental work on selecting labels, that was described above (task 2).

The idea developed in this paper is to apply fuzzy set theory to the raw data used in [1] and to compare the outcome of this process to the recommendations presented by [1]. The results are quite interesting and the fuzzy procedure shows such an admirable elegance that it could be of interest to the noise annoyance community.

In this paper the words "meaning", "exact meaning", "translate", and a few other terms relating to typical human activities are frequently used. The authors are fully aware that these are multidimensional constructs and that fuzzy set theory can (not yet) achieve the ambitious goal to imitate them. Hoping that the reader is fully aware of these limitations, we continue using these terms for simplicity.

This paper is organized as follows. Section 2 contains the fun bit for those readers interested in math and fuzzy set theory. The following sections can easily be read without bothering about this first section. Section 3 elaborates on translating linguistic terms used in noise annoyance surveys. We think it can be interesting for those readers that want to compare their results to their colleagues' in other parts of the world. Section 4 focuses on the selection of five-point scales. There the equi-vagueness is introduced as an additional requirement, especially if the results of the survey are going to be interpreted as equidistant fuzzy sets. Section 5 concludes with some thoughts on the impact of our findings on future work.

2. Mathematical background

2.1. Constructing membership functions

One of the techniques for gathering raw data to construct membership functions of linguistic terms consists in asking a sufficiently large group of test subjects to mark their idea concerning the meaning of a term presented to them, in the predefined universe U (e.g. on a continuous axis)



Figure 1. Probability distributions for 4 annoyance modifiers in English.

[8]. This is precisely the procedure used in the international scaling study [1]. The marks of the group of Ntest subjects now have to be aggregated in a suitable way or in other words a well-matched data reduction process must be applied [6]. Classical statistical analysis suggests to use a Gaussian probability distribution based on a calculated average and standard deviation. On a limited interval of values [0, 10], Gaussian distribution tends to fail to describe distributions that cluster near the end points of the scale. Asymmetric Gaussian bell shapes with different left and right standard deviation can solve this problem to some extent but they seem to underestimate the importance of tails in the distribution. Eventually a discretization using 11 equally distributed intervals was preferred. This number of intervals is sufficiently low to reduce measurement noise and sufficiently large to assure an accurate description of the membership functions. An additional advantage over piecewise linear approximations that are quite commonly used for describing membership functions, is the simplicity of numerical calculations. The fact that all tests and calculations seem to work quite well also validates this approximation.

As an example, Figure 1 shows distributions obtained directly from the survey for four English annovance modifiers. These distributions can be interpreted within the context of probability as the probability that an Englishspeaking person chooses a given number on the x-axis when asked to locate the modifier. Probability distributions in Figure 1 have been normalized accordingly. It is quite likely that these distributions will not become any narrower when more subjects are asked and more data are processed. This observation can to some extent be validated by the fact that these distributions resemble each other in different tests (for different languages) with different numbers of subjects. Therefore, what is expressed in the distribution is not an uncertainty on the determination of a well defined but unknown crisp value of the x-parameter, but vagueness inherently linked to the use of that particular linguistic term. With this interpretation in mind it is clear that the difference between "moderately annoyed" and "very annoyed" in Figure 1 is not merely a shift of the magnitude of annoyance, but "moderately an-

Term	English	Dutch
L01	0.63	0.50
L02	0.48	0.59
L03	0.51	0.43
L04	0.45	0.44
L05	0.42	0.39
L06	0.46	0.43
L07	0.47	0.46
L08	0.43	0.36
L09	0.43	0.37
L10	0.37	0.39
L11	0.47	0.41
L12	0.39	0.52
L13	0.41	0.42
L14	0.44	0.48
L15	0.41	0.50
L16	0.42	0.48
L17	0.51	0.53
L18	0.47	0.44
L19	0.49	0.38
L20	0.46	0.45
L21	0.46	0.56

Table I. Optimal α for all English and Dutch annoyance terms.

noyed" is also a term that is less vague. The advantage of fuzzy linguistic techniques is precisely that they allow to include this vagueness in the mathematical procedures used, for example in calculating the similarity between terms.

In fuzzy set theory, a linguistic term t is represented by a $U \rightarrow [0, 1]$ mapping A on the universe of discourse U. This mapping is interpreted as a possibility distribution, that is the value associated with each point u in the universe (on the annoyance axis) represents the possibility that the noise annoyance described by t is u. Several techniques have been proposed to translate a probability distributions, p, to a possibility distribution, π (or membership function) [9]. A numerically very easy transformation is the conservation of uncertainty method, which results in a transformation

$$\pi_i = \left(\frac{p_i}{\sup_j(p_j)}\right)^{\alpha},$$

where the index refers to the 11 intervals used in defining the probability distribution. This conversion is based on the principle of uncertainty conservation, stating that the amount of inherent uncertainty should be preserved when transformed from one theoretical description into another. For the probability distribution, the well-known Shannon entropy is used as uncertainty measure, defined by

$$H(p) = \sum_{i=1}^{n} p_i \log_2 p_i.$$

In [9] two types of uncertainty are identified, nonspecificity $N(\pi)$ and discord $D(\pi)$. The former measures the spread while the latter measures the ambiguity. To calcu-



Figure 2. Possibility distributions (membership functions) of 4 annoyance modifiers in English.

late N and D, it is assumed that the possibilities are ordered so that $\pi_1 \ge \pi_2 \ge \ldots \ge \pi_n$, which results in the definition

$$N(\pi) = \sum_{i=2}^{n} \pi_i \log_2\left(\frac{i}{i-1}\right),$$

and

$$D(\pi) = -\sum_{i=1}^{n-1} (\pi_i - \pi_{i+1}) \log_2 \left(1 - i \sum_{j=1+1}^n \frac{\pi_j}{j(j+1)} \right).$$

The total amount of uncertainty of a possibility distribution is given by their sum. This means that the principle of uncertainty conservation states that $H(p) = N(\pi) + D(\pi)$. It is conjectured that the mentioned log-scale interval transformation is the only one that exists for all distributions and is unique. The positive constant α is determined by minimizing the difference between H and N + D and lies in the interval [0, 1]. The optimal α were determined for 21 English and Dutch terms included in the study. Results are shown in Table I.

To ease calculations and because it can be observed from the table that the optimal α is mostly situated around the value of 0.5, this parameter will be fixed to 0.5 during all further computations. Figure 2 shows the possibility distributions associated with the same four English modifiers as used in Figure 1. Observe the way possibility distributions are normalized differently than probability distributions. The procedure described above is repeated for all 21 terms in the 9 languages available in the database. Let F(U) denote the collection of all these fuzzy sets related to linguistic terms.

2.2. Similarity measures

Based on the above observations it can be outlined that a good similarity measure should:

- Consider coincidence of the maximum of the membership functions,
- Consider the similarity in general shape of the membership functions.



Figure 3. Graphical comparison of membership functions of a few English terms to the Dutch term "tamelijk".

Table II. Similarity measures between "tamelijk gehinderd" and a few English terms.

Tamelijk	S_1	S_2	Eql_W	Sim
Partially	1.00	0.69	0.34	0.69
Somewhat	1.00	0.83	0.69	0.83
Fairly	0.86	0.85	0.68	0.85
Moderately	0.82	0.53	0.33	0.53
Rather	0.82	0.67	0.38	0.67

Mathematically speaking, a similarity measure on a universe U is a [0, 1]-valued indicator suitable for the comparison of fuzzy sets on U, i.e. a binary fuzzy relation on F(U), or stated otherwise, a fuzzy set on the universe $F(U) \times F(U)$. Depending on the requirements imposed on the measures, different indicators with varying behavior can be selected. Following Tsiporkova and Zimmermann [10], we make a basic distinction between measures inspired by set equality, and degrees of compatibility or overlap. A binary fuzzy relation Eql on F(U) is called a t-equality if $Eql(A, B) = 1 \Leftrightarrow A = B$, Eql(A, B) =Eql(B, A), and $T(Eql(A, B), Eql(B, C)) \leq Eql(A, C)$ are satisfied for any fuzzy sets A, B, and C on U, where T is any t-norm (for definition see Appendix A1). A reflexive, symmetric binary fuzzy relation Com on F(U) is called a degree of compatibility if it satisfies the condition

$$Com(A, B) = 0 \Leftrightarrow \sup_{u \in U} \min (A(u), B(u)) = 0$$

for any A and $B \in F(U)$.

In [11] an interesting class of t-equalities is studied in detail. They are defined by translating the logic relation that states that A and B are equal when A is a subset of B and at the same time B is a subset of A, into the equation

$$Eql_T(A, B) =$$

$$T\left(\inf_{u \in U} I_T(A(u), B(u)), \inf_{u \in U} I_T(B(u), A(u))\right)$$

for any A and B in F(U). T can be any t-norm and I_T is the associated residual implicator (defined in Appendix A1). The choice of t-norm in this expression is



Figure 4. Graphical comparison of membership functions of a few English terms to the Dutch term "enorm".

Table III. Similarity measures between "enorm gehinderd" and a few English terms.

Enorm	S_1	S_2	Eql_W	Sim
significantly very highly strongly severely	0.87 0.88 0.88 0.88 0.88 0.82	0.41 0.56 0.67 0.73 0.63	0.00 0.18 0.19 0.33 0.42	0.41 0.56 0.67 0.73 0.63

guided by the performance of the equality measure in the particular application context. For our application the Lukasiewicz t-norm W, defined by, for x and y in [0, 1], $W(x, y) = \max(0, x + y - 1)$ seems a good choice [7]. The influence of this choice on results is studied in detail in Appendix A2. The associated residual implicator is called the Lukasiewicz implicator, and can be expressed as $I_w(x, y) = \min(1, 1 - x + y)$ for x and y in [0, 1].

As degree of compatibility two measures S_1 and S_2 are considered. They are defined as

$$S_1(A,B) = \frac{\sup_{u \in U} T(A(u), B(u))}{\sup_{u \in U} S(A(u), B(u))}$$

and

$$S_1(A,B) = \frac{\sum_{u \in U} T(A(u), B(u))}{\sum_{u \in U} S(A(u), B(u))}$$

where *T* is a t-norm and *S* a t-conorm (see Appendix A1). S_1 compares peak regions of both fuzzy sets by assessing the height of their intersection. S_2 focuses on an overall overlap of the membership functions. Common choices for the t-norm and t-conorm in these compatibility measures are the operators as introduced by Zadeh, t-norm: $M(x, y) = \min(x, y)$ for x and y in [0, 1] and t-conorm: $M^*(x, y) = \max(x, y)$ for x and y in [0, 1].

To compare the performance of the equality and both compatibility measures presented above, a graphical representation of membership functions for a few English terms is compared to the Dutch term "tamelijk gehinderd" in Figure 3 and another set of English terms is compared

	German	French	Japanese	Spanish
not at all	überhaupt nicht	pas du tout	Mattakunai	absolutamente nada
insignificantly	kaum, ein wenig, wenig	presque pas	Hotondonai	apenas
barely	kaum	presque pas	Hotondonai	apenas
hardly	kaum	presque pas	Hotondonai	casi nada
a little	wenig	presque pas, guère	Amarinai, Taishitenai	escasamente
slightly	wenig	guère	Amarinai, Taishitenai	escasamente, un poco
partially	teilweise, einigermaßsen	légèrement	Sorehodonai, Wazukani, Sukoshi, Ikuraka	algo, un tanto
somewhat	einigermaßen	modérement, assez, plutôt	Sukoshi, Ikuraka, Yaya, Tashou	aprecciablemente
fairly	einigermaßen	plutôt	Ikuraka, Yaya, Tashou	aprecciablemente
moderately	Mittelmäßig	modérement	Ikuraka, Yaya, Tashou, Hikakuteki, Warini	moderamamente
rather	einigermaßen ziemlich	plutôt	Yaya, Tashou	aprecciablemente, bastante
considerably	ziemlich	plutôt	Hikakuteki, Warini	bastante, considerablemente
substantially	ziemlich	plutôt, beaucoup, vraiment	Hikakuteki, Warini	bastante, considerablemente
importantly	ziemlich, beträchtlich	plutôt, vraiment	Hikakuteki, Warini	bastante
significantly	ziemlich, beträchtlich, besonders	beaucoup, vraiment	Daibu	bastante considerablemente, muy
very	besonders, sehr	beaucoup, très	Daibu	muy, altamente
highly	stark, sehr, erheblich	très	Daibu	altamente, fuertemente
strongly	stark, sehr, erheblich	très	Daibu, Kanari	altamente, fuertemente
severely	äußerst	énormément	Hidoku	enormemente
tremendously	äußerst	énormément	Hidoku, Hijooni	enormemente
extremely	völlig	énormément	Hijooni	extremadamente

Table IV. Moderators with highest similarity to English terms in the first column for different languages.

to the Dutch term "enorm gehinderd" in Figure 4. Table II contains calculated similarities for "tamelijk" and Table III has the results for "enorm". Consider "tamelijk" first. The membership functions of "partially annoyed" and "somewhat annoyed" peak at exactly the same location as "tamelijk gehinderd" and hence S_1 shows a perfect match. This similarity measure fails to see however that there is quite a difference between "partially annoyed" and "tamelijk gehinderd" away from the peak. S_2 and Eql_W discover the very similar form of the membership function of "somewhat annoyed", "fairly annoyed", and "tamelijk gehinderd" and associate a larger similarity to those terms. These measures do not highlight the fact that "fairly annoyed" actually peaks at another location than "tamelijk gehinderd". For this example, S_1 on one hand clearly gives additional information over S_2 and Eql_W on the other hand. Also remark that the English term "moderately annoyed" does fairly bad on all similarity measures although its average value is quite close to the average value of "tamelijk gehinderd". Let us now consider "enorm". The membership functions for none of the English modifiers peaks at the same location as the membership function for "enorm gehinderd" so S_1 shows no perfect match in this case. Three terms score 0.88. S_2 indicates the membership function of "strongly annoyed" as the most similar one, clearly based on a good general agreement in form in the middle amplitude region. The preference of "strongly annoyed" over "highly annoyed" is not large and is determined by the somewhat higher value for the membership function in u = 10. Eql_W considers "severely annoyed" to be the English term most equal to "enorm gehinderd". "Strongly annoyed" obtains a lower score because of the large penalty given to the mismatch in u = 7 and u = 10.

It is clear from the above examples that no single best similarity measure can be found. Based on the findings of [10], where ways of combining the best of different worlds into a robust similarity indicator are outlined, we opted for the following generic hybrid measure (it is noted that several variations exist on this theme):

$$Sim(A, B) = T\left(Com_1(A, B), S\left(Eql(A, B), Com_2(A, B)\right)\right),$$

where Com_1 and Com_2 are degrees of compatibility, Eql_T is a t-equality, T is a t-norm and S is a t-conorm. The obvious choice here is to take $Com_1 = S_1$, $Com_2 = S_2$, and $Eql_T = Eql_W$. For T and S, the Zadeh operators are used (min/max). The similarity measure Sim is also

	Turkish	Norwegian	Hungarian	Dutch
not at all	hic degil	ikke	egyáltalán nem, nem	helemaal niet
insignificantly	degil, onemsiz olcude,	minimalt	alig, picit	nauwelijks
	cok az			
barely	onemsiz olcude, cok az	ubetydelig	alig	nauwelijks
hardly	onemsiz olcude	ubetydelig	alig	nauwelijks
a little	cok az	litt	kicsit, némileg	weinig, iets, lichtelijk
slightly	hafifce	litt	némileg, kevéssé	lichtelijk
partially	biraz, az cok, soyle boyle	noe, delvis	észrevehetoen, mérsékelten	enigzins, matig
somewhat	soyle boyle	delvis, endel	észrevehetoen, mérsékelten	tamelijk
fairly	soyle boyle	endel, forholdsvis	mérsékelten	tamelijk
moderately	orta derecede	middels	közepesen	matig, tamelijk
rather	soyle boyle, oldukca	ganske	jelentosen	tamelijk
considerably	oldukca, bayagi	temmelig	jelentosen, meglehetosen	behoorlijk
substantially	oldukca, bayagi	temmelig, betydelig	jelentosen, meglehetosen	behoorlijk, aanzienlijk
importantly	oldukca, epeyce	temmelig	jelentosen, meglehetosen	behoorlijk, aanzienlijk, veel
significantly	oldukca. epeyce	temmelig	jelentosen, meglehetosen, különösen	aanzienlijk, veel
very	epeyce, bayagi,	mye, meget	különösen, nagyonna,	erg
	cok, cok fazla		módfelett	
highly	cok fazla	meget	nagyonna	sterk, zeer, ernstig
strongly	cok fazla	meget	nagyonna, módfelett	zeer, ernstig
severely	fevkalade,	alvorlig	mértéktelenül	uitermate
	asırı derecede			
tremendously	asiri derecede,	alvorlig	mértéktelenül	ontzettend, uitermate
	ieci sekilde			
extremely	teci sekilde	alvorlig, voldsomt	vegtelenül, rettenetesen	extreem

Table IV.	Continuation.	Moderators with	highest similarit	v to English	terms in the	first column fo	r different l	anguages
				/				

reported in Tables II and III. In this case it follows S_2 , which was subjectively speaking the best measure of similarity anyhow. In fact when looking at all similarities between terms in different languages it can be observed that S_2 dominates results except for a few odd terms clustering mainly near the extremes of the scale.

It is clear that the analysis of the similarity of the membership functions involves various choices of operators. These choices may influence results. Appendix A2 contains sensitivity analysis for some parameters. It shows that although there are differences, the procedure is in general rather stable towards these choices.

3. Translating noise annoyance modifiers

Using the theory described above, mathematical translation tables can be constructed for all combinations of the 9 languages included in the database by simply calculating the similarity between all the membership functions involved. Although the tool developed to perform the calculations gives every desired combination of languages as an output, we limit the discussion in this paper to combinations involving English as one of the two languages. Similarity between all 21 terms in all languages considered and English are given on the Acustica united with Acta Acustica CD-ROM. Let us first consider the problem of translating a linguistic term or modifier using this table, a possible application being the translation of the result of noise annoyance survey to another language for publication or communication purposes. The similarity given in the translation table is not a binary value. In fact the result of a translation of a particular English word to another language can itself be regarded as a fuzzy set on the universe L containing all relevant terms (21 in this case) in the database, the calculated similarity being the membership degree. For all practical purposes one term or at least a small set of terms has to be selected. Several techniques can be used for this selection.

- 1. All terms with similarity above a predefined threshold s_0 are good translations.
- 2. The term with the highest similarity is the translation.
- 3. All terms within a range δ from the highest similarity s_{max} are good translations.

The first approach looks quite appealing at first sight but it does not always result in a translation using a limited vocabulary, as is the case for the noise annoyance modi-

fiers. Lowering s_0 does not solve this problem since this would result in too many translations for other words. The second approach must be rejected on the basis that it is to sensitive for measurement error in the determination of the membership functions. The third approach is used to translate the 21 English modifiers to 8 other languages $(\delta = 0.05)$. Results are shown in Table IV. The attention of the reader is drawn to some particular features in this table. It is for example easy to see that terms like "barely annoved" and "hardly annoved" are so close in meaning that they translate to the same terms in many of the other languages. The Japanese list of modifiers seems to contain many modifiers that are close in meaning to "partially", "somewhat", "fairly" and "moderately". Detailed analysis of the similarity values available on the CD-ROM shows that there is an important difference between translating "moderately" to Japanese and the three other terms. For "moderately" all similarities are low, meaning that a good fit can not be found in the Japanese database. Similarities for the other three terms mentioned is high and therefore one can state that there are several words in the Japanese database with approximately the same meaning. Translating "very" to Turkish shows similar problems.

A technique often used to check the quality of translation is to translate back to the original language and compare results. It is indeed known that linguistic translation is not a symmetric process. The procedure based on fuzzy sets proposed here is also not symmetric even though it takes into account less of the subtle nuances human translation does. Table V shows fuzzy-translation from English to Dutch and back, the final result being the accumulation of all English terms that can be found as translations of the Dutch terms in the second column. Typically this process results in an increase of alternatives, although this is not necessarily the case (e.g. "strongly"). The number of alternatives finally obtained depends strongly on the size of the vocabulary that is used and therefore it is not a good indicator of the quality of the translation process. It is however important that the original term is amongst the final list. This is not the case for "insignificantly", "moderately", and "rather". The exact fuzzy-meaning of these modifiers gets lost when translated to Dutch because there is no accurate translation for them in this language.

4. Selecting 5-point scale labels

4.1. Analysis of conclusions of the modifier study, based on fuzzy set theory

In [1] labels for a 5-point scale are proposed on the basis of a complex analysis that involves the rating on a continuous scale that is also used in this paper (*task 2*), the direct selection of 5 words by each subject (*task 3*), and input from noise annoyance experts. These last two factors introduce information on the preference of particular words in the context of describing noise annoyance that is not considered in the fuzzy approach (except for the fact that the original 21 modifiers are already chosen to be used in conjunc-

Table V. Translation from English to Dutch and back.

English	Dutch	English
not at all	helemaal	not at all
insignificantly	nauwelijks	barely, hardly
barely	nauwelijks	barely , hardly
hardly	nauwelijks	barely, hardly
a little	weinig, iets, lichtelijk	a little, slightly
slightly	lichtelijk	slightly
partially	enigzins, matig	partially, somewhat, fairly
somewhat	tamelijk	somewhat, fairly
fairly	tamelijk	somewhat, fairly
moderately	matig, tameliik	partially, somewhat, fairly
rather	tamelijk	somewhat, fairly
considerably	behoorlijk	considerably , substantially, importantly
substantially	behoorlijk, aanzienlijk	considerably, substantially , importantly, significantly
importantly	behoorlijk, aanzienlijk,	considerably, substantially, importantly , significantly
significantly	aanzienlijk, veel	substantially, importantly, significantly
very	erg	very
highly	sterk, zeer, ernstig	very, highly , strongly
strongly	zeer, ernstig	strongly
severely	uitermate	severely, tremendously
tremendously	ontzettend, uitermate	severely, tremendously
extremely	extreem	extremely

tion with annoyance). The similarity between noise annoyance modifiers, calculated using fuzzy set theory, can be used to analyze the correspondence between these labels. Table VI shows the results. Similarity is not perfect (=1) but in most cases reasonably high. However, some labels in particular languages perform rather poor. They are highlighted in Table VI. The third label in Japanese compares very poorly to the third label in most other languages. A comparable problem emerges for the third label in Dutch. This label does correspond to the third label in Japanese however. Thus Japanese and Dutch form a separate group when looking at the middle label on the 5- point scale. French and Spanish conflict slightly with German for the fifth and highest label on the scale.

		Ger	Eng	Fre	Jap	Spa	Tur	Nor	Hun	Dut	ide
German	überhaupt nicht	1.00	0.93	1.00	0.85	1.00	0.76	0.91	0.90	1.00	0.56
	etwas	1.00	0.79	0.81	0.80	0.70	0.67	0.72	0.65	0.70	0.61
	mittelmäßig	1.00	0.74	0.72	0.33	0.77	0.78	0.89	0.89	0.37	0.65
	stark	1.00	0.73	0.66	0.75	0.66	0.68	0.64	0.77	0.71	0.62
	äußerst	1.00	0.71	0.41	0.76	0.49	0.63	0.56	0.64	0.53	0.69
English	not at all	0.93	1.00	0.93	0.85	0.93	0.84	0.91	0.90	0.93	0.57
Ũ	slightly	0.79	1.00	0.74	0.72	0.64	0.56	0.82	0.77	0.85	0.50
	moderately	0.74	1.00	0.52	0.49	0.63	0.84	0.67	0.87	0.53	0.63
	very	0.73	1.00	0.74	0.88	0.73	0.71	0.76	0.73	0.82	0.70
	extremely	0.71	1.00	0.58	0.91	0.72	0.75	0.74	0.80	0.70	0.82
French	pas du tout	1.00	0.93	1.00	0.85	1.00	0.76	0.91	0.90	1.00	0.56
	légèrement	0.81	0.74	1.00	0.77	0.66	0.62	0.71	0.60	0.73	0.62
	moyennement	0.72	0.52	1.00	0.29	0.69	0.57	0.70	0.65	0.33	0.68
	beaucoup	0.66	0.74	1.00	0.67	0.71	0.63	0.83	0.75	0.80	0.77
	extremement	0.41	0.58	1.00	0.57	0.88	0.57	0.75	0.77	0.88	0.04
Japanese	Mattakunai	0.85	0.85	0.85	1.00	0.85	0.83	0.95	0.95	0.85	0.65
	Sorehodonai	0.80	0.72	0.77	1.00	0.77	0.79	0.62	0.57	0.66	0.53
	Tashou	0.33	0.49	0.29	1.00	0.29	0.46	0.32	0.37	0.70	0.44
	Daibu	0.75	0.88	0.67	1.00	0.72	0.79	0.76	0.76	0.79	0.60
	Hijooni	0.76	0.91	0.57	1.00	0.69	0.72	0.70	0.79	0.69	0.81
Spanish	absolutamente nada	1.00	0.93	1.00	0.85	1.00	0.76	0.91	0.90	1.00	0.56
	ligeramente	0.70	0.64	0.66	0.77	1.00	0.68	0.63	0.51	0.64	0.56
	medianamente	0.77	0.63	0.69	0.29	1.00	0.73	0.86	0.74	0.33	0.68
	muy	0.66	0.73	0.71	0.72	1.00	0.73	0.80	0.68	0.82	0.68
	extremadamente	0.49	0.72	0.88	0.69	1.00	0.65	0.83	0.84	0.90	0.73
Turkish	hic degil	0.76	0.84	0.76	0.83	0.76	1.00	0.87	0.88	0.76	0.64
	biraz	0.67	0.56	0.62	0.79	0.68	1.00	0.54	0.45	0.56	0.51
	orta derecede	0.78	0.84	0.57	0.46	0.73	1.00	0.78	0.79	0.53	0.74
	COK	0.68	0.71	0.63	0.79	0.73	1.00	0.73	0.75	0.79	0.51
	lect sekilde	0.05	0.75	0.37	0.72	0.05	1.00	0.04	0.78	0.08	0.89
Norwegian	ikke	0.91	0.91	0.91	0.95	0.91	0.87	1.00	0.99	0.91	0.61
	litt	0.72	0.82	0.71	0.62	0.63	0.54	1.00	0.69	0.84	0.57
	middels	0.89	0.67	0.70	0.32	0.86	0.78	1.00	0.85	0.35	0.65
	voldsomt	0.64	0.76	0.85	0.76	0.80	0.75	1.00	0.08	0.88	0.71
	volusonit	0.50	0.74	0.75	0.70	0.05	0.04	1.00	0.70	0.70	0.71
Hungarian	egyáltalán nem	0.90	0.90	0.90	0.95	0.90	0.88	0.99	1.00	0.90	0.62
	kissé	0.65	0.77	0.60	0.57	0.51	0.45	0.69	1.00	0.72	0.41
	nagyonna	0.89	0.87	0.05	0.37	0.74	0.79	0.85	1.00	0.42	0.62
	rettenetesen	0.77	0.73	0.73	0.70	0.08	0.73	0.08	1.00	0.90	0.38
Dutch	helemaal niet	1.00	0.03	1.00	0.85	1.00	0.76	0.01	0.00	1.00	0.56
Duici	een beetie	0.70	0.93	0.73	0.85	0.64	0.70	0.91	0.90	1.00	0.50
	tameliik	0.37	0.53	0.33	0.70	0.33	0.53	0.35	0.42	1.00	0.30
	erg	0.71	0.82	0.80	0.79	0.82	0.79	0.88	0.77	1.00	0.66
	extreem	0.53	0.70	0.88	0.69	0.90	0.68	0.76	0.90	1.00	0.76
ideal	term 1	0.56	0.57	0.56	0.65	0.56	0.64	0.61	0.62	0.56	1.00
	term 2	0.61	0.50	0.62	0.53	0.56	0.51	0.57	0.41	0.56	1.00
	term 3	0.65	0.63	0.68	0.44	0.68	0.74	0.65	0.62	0.49	1.00
	term 4	0.62	0.70	0.77	0.60	0.68	0.51	0.71	0.58	0.66	1.00
	term 5	0.69	0.82	0.64	0.81	0.73	0.89	0.71	0.87	0.76	1.00

Fable VI. Similarity between lab	cels proposed for	a 5-point scale	in [1], labels in italic	were changed by the a	uthor of [1].
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	label 1	label 2	label 3	label 4	label 5
German	nicht	etwas, teilweise	mittelmäßig	beträchtlich, beson- ders, stark	völlig
English	insignificantly	slightly, partially	moderately	very, strongly	extremely
French	pas	légèrement	moyennement	beaucoup	énormément
Japanese	Hotondonai	Amarinai, Taishite nai, Sorehodonai	Yaya, Tashou , Hikakuteki, Warini	Daibu	Hijooni
Spanish	insignifi- cantmente	un poco, algo, un tanto	medianamente	muy , altamente	extremadamente
Turkish	degil	hafifce, birazcik, bir miktar, biraz , az cok	orta derecede	epeyce, cok fazla	feci sekilde
Norwegian	minimalt	noe	middels	mye	alvorlig
Hungarian	egyáltalán nem , nem, alig	mérsékelten	közepesen	nagyonna	rettenetesen
Dutch	niet	iets, lichtelijk, een beetje , enigzins, matig	matig, tamelijk , behoorlijk	erg, sterk	extreem

Table VII. Best match with the 5 fuzzy ideal labels in each of the languages considered.



Figure 5. Membership functions for an ideal fuzzy division of the universe of degrees of annoyance.

4.2. Choice of 5-point scale labels based on fuzzy ideal language

This work is part of a larger effort to use fuzzy rule based systems to calculate noise annovance [12, 13]. In fuzzy set theory a complete set of membership functions is preferred to subdivide a universe. Figure 5 shows a typical set of 5 triangular membership functions often used in fuzzy set theory. It can be argued that a language containing words that can be represented by such a set of membership functions would be ideal to label a five-point scale if results of a survey are to be used in (fuzzy) modeling. The modifiers are not only equidistant on the annoyance scale but also have the same degree of vagueness. Most analyses and modeling efforts have assumed that the verbal adjectives were equally spaced so guaranteeing this property as much as possible is essential anyhow. The 5 labels constructed in the ideal language can now be translated to the 9 natural languages considered in this paper using the fuzzy similarity approach. In this process all 21 terms from the modifier study are considered. Figure 6 gives the similarity for the best match in each language for each of the 5 ideal labels. For most languages, similarity is better for



Figure 6. Similarity of the best fit of terms in all languages considered to the mathematically ideal 5-point scale labels.

the first and the last label. The middle label seems hard to translate to Japanese and Dutch within the available vocabulary. The second label translates somewhat less easy to Turkish, Japanese, and English.

Table VII lists the best matches for each language, again taking into account a margin $\delta = 0.05$. Terms corresponding to the 5-point scale labels considered in Table VI are shown in bold. In most languages 3 to 4 terms are recovered. It is striking that the first label does not seem to correspond to the label proposed in [1] for any of the languages except Hungarian. Note that precisely this label was predetermined in tasks 3 and 4.

5. Discussion and conclusions

In fuzzy set theory, linguistic approximation plays an important role since one of the goals of this theory is to explore so called calculation with words. Similarity measures that are used to compare fuzzy sets allow to take into account not only the average location of two sets in the universe, but also open the opportunity to include a similar degree of vagueness as part of a measure of similarity. This opportunity is explored in an effort to re-analyze data from the international comparison of noise annoyance modifiers. This leads to some interesting conclusions.

Translation of annoyance modifiers based on the fuzzy set theoretical approach presented here, gives a list of words in several languages for which it can be argued that it is better than the average translation found in a dictionary since it is derived in the very specific context of noise annoyance. Therefore one could argue that it should shed some light on the interpretation of the outcome of annoyance surveys performed in another language than the one a researcher is familiar with. Moreover a quality indication is given for each possible translation. This results amongst others in a number of equally valid translations for particular words. The reader should be conscious however that this mathematical translation neither takes into account how commonly a combination of words is used in a language nor the fact that words may have unwanted connotations in the noise annoyance context. These factors are taken into account in the analysis presented in [1]. For the purpose of grasping the subtle difference between modifiers used in different languages in scientific analyses of data the advantage of considering the vagueness in the modifier can be of greater importance than the drawback mentioned above.

One should be careful to bear in mind that translating is not a symmetric operation. Translating a modifier from one language to another and back does not necessarily result in the original modifier. This observation still holds when translation is simplified to a (fuzzy) mathematical process as is done in this work. In the best case, the result of back-translation is a list of equally possible modifiers, which contains the original word. If not, the vocabulary considered may be too limited (21 words in this case) or there may just not be a good translation in the other language. Such words may be excluded from documents where international standardization is the issue. When more languages are combined this could however result in a fairly limited set of candidate words for this standardization. A possible alternative, not elaborated on in this work could be to combine modifiers using an OR operation. The procedure used in [1] resulted in a proposal for internationally standardized 5-point and 4-point scale labels. The fuzzy set theoretical approach presented in this paper adds a new dimension, the degree of vagueness to the analysis. From the point of view of fuzzy modeling of the outcome of surveys conducted using these labels, this is an important factor. However it may also be of importance to consider this vagueness when one tries to explain

differences between knowledge extracted from surveys in different language regions. Fortunately the fuzzy analysis confirms that in almost all languages the modifiers proposed in [1] are sufficiently similar in vagueness. A few exceptions are highlighted. The Japanese middle of the 5-point scale term, but also the middle of the scale term in Dutch seem to correspond not very well to the middle of the scale terms in other languages. French and Spanish conflict slightly with German for the fifth and highest label on the scale.

A possible way to introduce the same amount of vagueness in all labels of a say 5-point scale starts from a set of labels that is ideal from a fuzzy logic point of view and translate these terms from the ideal language to the natural languages considered. This exercise results in poor similarity for one or two of the labels in Dutch, Japanese, Turkish and English. This implies that other words need to be added to the vocabulary or that a combination of modifiers may be required to describe a set of labels that is ideal for fuzzy rule based modeling. Neglecting this fact, a short-list of candidates for 5-point scale labels can be obtained (Table VII). Fuzzy set theory is indecisive concerning the final selection from this short-list so additional components must be added. This was done in [1]. In most of the languages considered the selection made in [1] is indeed in the list proposed by fuzzy set theory. The elegance with which fuzzy set theory comes to this selection is at least admirable. The selection for the first label rarely contains the label proposed in [1]. This should draw our attention. Looking more carefully at the possibility distributions learns that the amount of vagueness in the first label is much more similar to the amount of vagueness in the other labels on the scale than when the labels from [1] are used. In other words, the lowest label proposed in [1] may be focussed too much on the extreme of the annoyance-modifying universe for fuzzy rule based modeling purposes. The very crisp definition of the lowest label may lead to a binary decision between no annoyance and a certain degree of annoyance that bears some resemblance to using a filter question preceding the level of annoyance question. In analyzing results based on the 5 labels proposed in [1] it is worthwhile to think about the data as if they were obtained with a questionnaire including such a filter question. It is not clear whether the difference in the amount of vagueness in the terms used to label the scale can influence the choice test subjects make from the n-point scale presented to them. A tendency that could be expected is that more vague labels tend to attract while more crisp labels tend to repel. There is however no evidence to confirm this hypotheses.

As a final remark the authors would like to stress that the analysis above starts from the premises that the word used to describe annoyance itself has exactly the same meaning and carries the same amount of vagueness in all languages considered. This does not influence any of the conclusions drawn above if and only if the modifiers are used in conjunction with the word for annoyance that was used in the modifier study.

Name	Norm	Conorm	Residual Implicator
Zadeh	$M(x,y) = \min(x,y)$	$M^*(x,y) = \max(x,y)$	$I_M(x,y) = \begin{cases} 1 & (\text{if } x \leq y); \\ y & (\text{if } x > y). \end{cases}$
Product	$P(x,y) = x \cdot y$	$P^*(x,y) = x + y - x \cdot y$	$I_P(x, y) = \begin{cases} 1 & \text{(if } x \leq y); \\ y/x & \text{(if } x > y). \end{cases}$
Lukasiewicz	$W(x,y) = \max(0, x+y-1)$	$W^*(x,y) = \min(1,x+y)$	$I_W(x, y) = \min(1, 1 - x + y)$

Table A1. Summary of common fuzzy t-norm and their associated t-conorm and residual implicator.

Table A2. Comparison of selection of 5 labels for different choices of operators defined in Table A.I. The first column indicates operators used for S_1 , S_2 , S_{eql} and Sim.

	label 1	label 2	label 3	label 4	label 5
MMM M	niet	een beetje, lichtelijk, enigzins, matig, iets	tamelijk, matig, behoorlijk	sterk, erg	extreem
MMW M	niet	een beetje, lichtelijk, enigzins, matig, iets	tamelijk, matig, behoorlijk	sterk, erg	extreem
WWW M	niet, helemaal niet	enigzins	behoorlijk, tamelijk, matig	erg	extreem
MMM P	niet	een beetje, lichtelijk, enigzins, matig	matig	sterk, erg, zeer, veel	extreem
MMW P	niet	enigzins	matig, behoorlijk, tamelijk	erg, veel, sterk	extreem
WWW P	niet, helemaal niet	enigzins	behoorlijk, matig	erg, veel	extreem
MMM W	niet	een beetje, lichtelijk, enigzins, matig	matig	sterk, erg	extreem
MMW W	niet, helemaal niet	enigzins	matig	veel, erg, sterk, ernstig, zeer	uitermate, extreem
www.w	niet, helemaal niet	enigzins	matig, behoorlijk	erg	extreem, uitermate

Appendix

A1. Fuzzy operators

In this article a fuzzy set A is introduced as a mapping from the universe, U, into the unit interval [0, 1], called the membership function, μ_A . The membership function can be seen as an extension of the characteristic function, χ_C , that characterizes a classical (crisp) set C, which is a mapping $U \rightarrow \{0,1\}$. When $\chi_C(u) = 1$, then $u \in C$, else when $\chi_C(u) = 0$, $u \notin C$. By allowing any value in the unit interval, this "membership value" can gradually transform from not belonging to the set to full membership [14].

In the same spirit, the classical operations on crisp sets are also extended. A generalization of the intersection operation, which corresponds to the AND-operation in logic, is known as a triangular norm (also called tnorm). A t-norm, T, is a symmetric, associative, increasing $[0,1] \times [0,1] \rightarrow [0,1]$ mapping satisfying T(1,x) = xfor every $x \in [0,1]$. Please observe that when restricted to $\{0,1\}$ this mapping coincides with the truth table of the classical AND operator. The dual operation of a tnorm is called a triangular conorm, or t-conorm. A tconorm, S, is defined as a symmetric, associative, increasing $[0,1] \times [0,1] \rightarrow [0,1]$ mapping satisfying S(0,x) = xfor every $x \in [0,1]$. A t-conorm extends the union operation on sets and the OR operator in logic. In fuzzy literature, several choices for norms and conorms exist, Table A1 shows some common examples, including the original operators proposed by Zadeh.

The following ordering can be proven: $W \leq P \leq M$ (largest norm) $\leq M^*$ (smallest conorm) $\leq P^* \leq W^*$ [15].



Figure A1. Similarity between five English annoyance modifiers and the Dutch term "tamelijk" as a function of t-norm used in the definition of the similarity measures; M=Zadeh, P=product, W=Lukasiewicz.

Not only the AND and OR logical operators can be "fuzzified", also the classical implicator can be extended. Among several families of fuzzy implicators that were proposed, the residual implicators are widely used in applications. A residual implicator I_T is defined as $I_T(x, y) = \sup\{\gamma \in [0, 1] | T(x, \gamma) \leq y\}$, for each x and y in [0, 1]. Also see Table A1 for some common incarnations.

A2. Sensitivity analysis

The definition of the t-equality Eql_T , the degrees of compatibility S_1 and S_2 and the combined similarity measure Sim used in this work all depend upon the choice of a t-norm. Note that once the t-norm is chosen we can use its associated t-conorm and residual implicator (see Table A1). There are no strict guidelines for choosing these operators and the optimal choice of operators to construct similarity measures is known to be context and application-dependent [10]. Therefore this appendix considers the dependence of a few main conclusions on the choices made. In the bulk of the paper the Zadeh operators were proposed for S_1 , S_2 and Sim, and the Lukasiewicz operator was used for Eql_T . The dependence of similarity on the choice of operators is illustrated in Figures A1 and A2 for similarity between two Dutch terms and a set of English terms. Similarities mainly change a lot in magnitude but the order of degrees of similarity between terms changes rarely. The analysis of the impact on final results is limited to the more elaborate task of selecting 5 labels corresponding to the ideal fuzzy triangular membership functions. It is also limited to one language. Table A2 shows the results. A number of changes in the proposed lists can be observed so this result is indeed sensitive to the choice of operators. However, concerning the conclusions of this work two facts stand out. Firstly, for each label at least one particular term shows up for all choices of operators. Secondly, one of the main conclusions concerning the choice of "niet" rather than "helemaal niet" as the first label seems to stand particularly firm.



Figure A2. Similarity between five English annoyance modifiers and the Dutch term "enorm" as a function of t-norm used in the definition of the similarity measures; M=Zadeh, P=product, W=Lukasiewicz.

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