# Efficient Approximate Reasoning with Positive and Negative Information

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**Abstract.** Starting from the generic pattern of the Generalized Modus Ponens, we develop an efficient yet expressive quantitative model of approximate reasoning that tries to combine "the best of different worlds"; following a recent trend, we make a distinction between *positive* or observed ("guaranteed") fuzzy rules on one hand, and *negative* or restricting ones on the other hand, which allows to mend some persistent misunderstandings about classical inference methods. To reduce algorithm complexity, we propose inclusion–based reasoning, which at the same time offers an efficient way to approximate "exact" reasoning methods, as well as an attractive implementation to the concept of reasoning by analogy.

**Keywords:** approximate reasoning, positive and negative information, possibility theory, inclusion measures

## 1 Introduction and Motivation

Reasoning with imprecise information expressed as fuzzy sets (possibility distributions) has received much attention over the past 30 years. More specifically, researchers have undertaken various attempts to model the following reasoning scheme (an extension of the modus ponens logical deduction rule), known as Generalized Modus Ponens (GMP):

$$\begin{array}{ll} \textbf{IF } X \text{ is } A & \textbf{THEN } Y \text{ is } B \\ \quad X \text{ is } A' & \\ \hline & Y \text{ is } B' \end{array}$$

where $X$ and $Y$ are assumed to be variables taking values in the respective universes $U$ and $V$; furthermore $A, A' \in \mathcal{F}(U)$ and $B, B' \in \mathcal{F}(V)$[1].

[1] By $\mathcal{F}(U)$ we denote all fuzzy sets in a universe $U$, i.e. mappings from $U$ to $[0, 1]$.

M.Gh. Negoita et al. (Eds.): KES 2004, LNAI 3214, pp. 779–785, 2004.

Traditionally, the if–then rule is represented by a fuzzy relation $R$ (a fuzzy set in $U \times V$), and to obtain an inference $B'$ about $Y$, the direct image $R' \uparrow_{\mathcal{T}} A$ of $A'$ under $R$ by means of a t–norm[2] $\mathcal{T}$ is computed[3], i.e. for $v$ in $V$,

$$B'(v) = R \uparrow_{\mathcal{T}} A'(v) = \sup_{u \in U} \mathcal{T}(A'(u), R(u,v)) \tag{1}$$

$R$ is typically modelled by either a t–norm $\mathcal{T}$ or an implicator[4] $\mathcal{I}$, such that for all $u$ in $U$ and $v$ in $V$

$$R(u,v) = \mathcal{T}(A(u), B(v)) \tag{2}$$

$$\text{or,} \quad R(u,v) = \mathcal{I}(A(u), B(v)) \tag{3}$$

This choice gives rise to the *conjunction–based*, resp. *implication–based* model of approximate reasoning (see e.g. [1]). Also (1) can be easily generalized to a batch of parallel fuzzy rules (as in a fuzzy expert system); in this paper we do not consider this extended setting.

Two important points should be made w.r.t. this “de facto” procedure:

1. Regarding *semantics*, Dubois et al. [4] recently pointed out that when $R$ is modelled by a t–norm as in (2), the application of (1) invokes undesirable behaviour of the reasoning mechanism.
2. Regarding *complexity*, the calculation of the supremum in (1) is a time–consuming process. When $|U| = m$ and $|V| = n$, the complexity of a single inference amounts to $\mathcal{O}(mn)$.

We are convinced that these arguments can be identified as the main causes why the application of approximate reasoning has been restricted so far to simple control tasks, and why only crisp numbers are used as input values to the GMP (as in Mamdani controllers). In this paper, starting from the distinction between positive and negative information in the light of possibility theory (Section 2), in Section 3 we present a unified reasoning mechanism that takes into account a rule's intrinsic nature. Section 4 tackles the efficiency issue: we show that inclusion–based approximate reasoning, as a natural tool for reasoning by analogy, may reduce complexity to $\mathcal{O}(m+n)$ without harming the underlying rule semantics.

## 2 Positive and Negative Information in Possibility Theory

Possibility theory is a formalism that tries to capture in mathematical terms imprecise (typically, linguistic) information about the more or less plausible values

[2] A t–norm $\mathcal{T}$ is an increasing, commutative, associative $[0,1]^2 \to [0,1]$ mapping that satisfies $\mathcal{T}(x,1) = x$ for all $x$ in $[0,1]$.

[3] This procedure is also known as Compositional Rule of Inference (CRI).

[4] An implicator $\mathcal{I}$ is a $[0,1]^2 \to [0,1]$ mapping with decreasing first and increasing second partial mappings that satisfies $\mathcal{I}(0,0) = 1$ and $\mathcal{I}(1,x) = x$ for all $x$ in $[0,1]$.